

FTUAM-99/02; IFT-UAM/CSIC-99-02

Recent Developments in Physics Far Beyond the Standard Model ¹

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Abstract

String theory is the leading candidate for a unified theory of the standard model and gravity. In the last few years theorists have realized that there is a unique structure underlying string theory. In this unification a prominent role is played by the duality symmetries which relate different theories. I present a brief overview of these developements and discuss their possible impact in low-energy physics. One of the lessons learned is that the string scale, usually asumed to be of order of the Planck scale, could be arbitrarily low, even close to accelerator energies.

¹Contribution to the proceedings of the XXVI International Meeting on Fundamental Physics, La Toja, Spain (June 1998).

1 Far Beyond the Standard Model (FBSM)

You have already heard many times that the Standard Model (SM) is fine and describes everything (except, maybe, neutrino masses) very well. And you have also heard almost the same number of times that in spite of that there are plenty of aspects of the SM that we do not yet understand. I am not going to repeat the well known list of questions without answers in the SM, it has not changed very much in the last decade. Instead of that I am going to talk today about one particular problem: the problem of combining the SM interactions with gravity. As is well known naive quantization of Einstein's gravitation leads to a non-renormalizable theory plagued with ultraviolet divergences. This problem has been with us for several decades and since fifteen years ago it is believed that string theory is the best candidate for a solution. In addition we need a theory in which chiral gauge interactions like those of the SM can coexist with quantum gravity. Also this aspect seems to be present in strings so let me first of all discuss some key ingredients of these theories.

2 String Theories

The origin of ultraviolet divergences in perturbative gravity (or any field theory for that matter) is related to the existence of interactions between particles which take place at a point. Insisting in pointlike interactions leads to ultraviolet divergences. Whereas in the field theories of the $SU(3) \times SU(2) \times U(1)$ standard model those divergences can be absorbed in the renormalization process, that is not the case for gravity. String theories bypass this problem by avoiding point-like interactions and allowing the particles to interact in an extended region of space-time.

String theory [1] is a radical departure from the standard understanding of physics because assumes that elementary (pointlike) particles are not the fundamental blocks of nature. Rather the fundamental objects are strings, extended one-dimensional objects. Whereas the movement of a classical particle is described by giving the dependence of coordinates on e.g., proper time τ , $X^\mu(\tau)$, the movement of a string in space-time is described by giving $X^\mu(\tau, \sigma)$, where σ is the coordinate along the string. Strings may be either open (with two boundary points X_a, X_b between which the string stretches) or closed ($X_a = X_b$). Interactions of strings take place by joining and splitting. Thus e.g., a closed string propagating in space can in a given moment split into two closed strings. In such a process of splitting the interaction takes place in an extended region in the variables (τ, σ) and this is at the basis of the absence of ultraviolet divergences in string theory.

An important property of string theory is that it predicts the existence of gravity. This comes about as follows. Let us consider the quantization of a closed string. One can consider this system as a set of an infinite number of

harmonic oscillators with two type of vibration modes, right-handed (say) and left handed around the closed string. If a_n^μ and \tilde{a}_n^μ ($n = 0, \pm 1, \pm 2, \dots$) are the two type of oscillators one finds for the masses of the string modes:

$$m^2 = \frac{1}{\alpha'}(N + \tilde{N} - 2) \quad (1)$$

where $N = \sum_n a_{-n}^\mu a_n^\mu$ and $\tilde{N} = \sum_n \tilde{a}_{-n}^\mu \tilde{a}_n^\mu$ are the corresponding number operators. Here $\frac{1}{\alpha'}$ is the string tension and the -2 comes from the (regulated) zero point vacuum energy. There is an additional constraint (coming from the non-existence of a privileged point on the string coordinate σ) which imposes $N = \tilde{N}$ for the eigenvalues of physical states. Now, for $N = \tilde{N} = 1$ there is a *massless state with two vector indices*, given by $g^{\mu\nu} = a_1^\mu \tilde{a}_1^\nu |0\rangle$. Thus we have a massless spin two state in the spectrum, which is no other but the graviton². Unfortunately for $N = \tilde{N} = 0$ we have also a tachyonic state with mass² equal to $-2/\alpha'$. This is signaling an instability in this (purely bosonic) string. Furthermore this theory has no fermions in the spectrum and hence it cannot accomodate the observed quark and leptons.

Both problems are solved if we supersymmetrize the string variable $X^\mu(\tau, \sigma)$ and include fermionic coordinates $\psi^\mu(\tau, \sigma)$. There are two types of closed superstrings of this type which go under the names of Type IIA and Type IIB [2]. They differ in the structure of the supersymmetry in both theories. The pairs X^μ, ψ^μ may be considered themselves as fields in a two-dimensional field theory on the variables (τ, σ) , the world-sheet. Both Type IIA and Type IIB have $N = 2$ supersymmetry on the worldsheet but the two SUSY generators have opposite chirality in the Type IIA case and the same chirality for Type IIB. In both cases one can show that, in an explicitly supersymmetric formalism the tachyon disappears from the massless spectrum. Furthermore they are finite theories in perturbation theory without any divergence, neither ultraviolet nor infrared. Thus they seem to provide us for the first time with consistent theories of quantum gravity. However they have a puzzling property: they are formulated in ten (one timelike and nine spacelike) dimensions. Extra dimensions are not necessarily a problem since, as already Kaluza and Klein showed in the twenties, they can be curled up into a compact space at very short distances in such a way that we only are able to see experimentally the standard four dimensions. In the case of Type II strings, six dimensions are assumed to be compactified at very short distances R in such a way that only if we have energies higher than $1/R$ we would be able to see the six extra dimensions. Consistency with known gravitational interactions suggest to identify :

$$1/R \propto \frac{1}{\sqrt{\alpha'}} \propto M_{Planck}. \quad (2)$$

²More specifically, it is the symmetrized state which gives rise to the graviton. The antisymmetrized state gives rise to an additional antisymmetric field $A^{\mu\nu}$ and the trace to a massless scalar, the dilaton ϕ .

The idea is that the SM particles (plus the graviton) correspond to string modes which are (approximately) massless compared to such huge mass scales.

Even compactifying the six extra dimensions, Type II theories do not seem very promising (at least in their pre-1995 formulation) to include not only gravity but the standard model. We know that one of the most important properties of the SM is its chirality: left-handed and right-handed quarks and leptons transform differently under the SM group. Type IIA theory is non-chiral and does not seem possible to include a chiral theory like the SM inside it. Type IIB theory is on the other hand chiral but it turns out that it does not lead upon compactification to non-abelian chiral theories like $SU(3) \times SU(2) \times U(1)$, it leads at most to chiral $U(1)$ theories.

Before 1984 only a third type of supersymmetric string, also defined in ten dimensions, was known in addition to Type II theories. It was the so called Type I theories. This theory (unlike Type II) contain both closed and open strings. Closed (unoriented) strings give rise to the gravitational sector of the theory and open strings give rise in general to non-Abelian gauge theories. This is a very interesting property since one then has the hope of embedding the gauge group $SU(3) \times SU(2) \times U(1)$ into the open string sector. Unfortunately it was realized [3] that the theory has anomalies and hence it is inconsistent at the quantum level. Thus before 1984 the situation concerning string theory was very puzzling: Type II theories had a consistent quantum theory without anomalies but where unable to embed the SM. On the other hand Type I strings had the potential to embed the SM but were anomalous! One thus can understand why before 1984 only very, very few theorists were working in the field of string theory.

In 1984 everything changed when it was realized that the conclusion that Type I string is inconsistent was in fact not true. Green and Schwarz showed [4] that for a very particular gauge group, $SO(32)$, all anomalies cancelled via a new and elegant mechanism. Soon after Gross, Harvey, Martinec and Rohm [5] showed the existence of two more string theories in ten dimensions, the heterotic strings. They are theories that, like Type I, are chiral and contain gauge fields in their massless spectra. However they only contain closed strings. The gauge groups are either $SO(32)$ or $E_8 \times E_8$ and are also anomaly free. These developments caused the well known increase of popularity of this field. Now we had three string theories which were candidates to unify gravitation and the SM into a finite theory. Of course, being ten-dimensional a process of compactification down to the four physical dimensions is required. Most phenomenological attempts to embed the SM were based in the heterotic $E_8 \times E_8$ string since, as shown by Candelas, Horowitz, Strominger and Witten [6], there are particular classes of compact six-dimensional varieties (known as Calabi-Yau manifolds) such that, upon compactification of the $E_8 \times E_8$ string give rise at low energies to an E_6 gauge theory with one $N = 1$ supersymmetry (useful to solve the hierarchy problem) and a number of fermion generations given by one half the Euler characteristic of the Calabi-Yau manifold. The gauge group

E_6 had been used as a grand unification group in the past so everything looked like if we were really close to the final unified theory of the standard model and gravity.

This optimism was cooled down when people tried to get a more specific contact with low energy phenomenology. It was soon realized that although E_6 is a nice group, is still far away from what we really want : the SM with three quark-lepton generations, appropriate Higgs doublets, a reasonable fermion mass spectrum, sufficiently stable proton, etc. A number of techniques (orbifolds, fermionic construction etc.) were developed to get miriads of new possibilities for heterotic string compactifications leading at low energies to plenty of possibilities for gauge group, fermion content and phenomenology. A few of the new string vacua obtained with the new techniques had massless spectrum quite close to that of the minimal supersymmetric SM (MSSM), which is already by itself a remarkable achievement [7]. In spite of this success, the presence of such miriads of apparently consistent string vacua leads to an obvious question: how is the physical vacuum (presumably corresponding to the SM) chosen by the dynamics of the theory? With unbroken supersymmetry all string vacua are degenerate, which leads us to the next question: how is supersymmetry broken in string theory? Both questions seem to require a knowledge of non-perturbative effects in string theory.

3 T-duality

By 1985 the five existing supersymmetric $D = 10$ string theories had already been discovered: Type IIA, Type IIB, $SO(32)$ Type I, $SO(32)$ heterotic and $E_8 \times E_8$ heterotic. As we said, much effort was dedicated to the study of the heterotic strings and also to the study of two-dimensional conformal field theory (CFT) in general, the latter being relevant to study the properties of the string variables $X(\tau, \sigma)$ in the two-dimensional world-sheet spanned by τ and σ . Soon a peculiar symmetry of closed strings was noticed, which now goes under the name of T-duality. Consider now for simplicity the case of the purely bosonic string with coordinates $X^\mu(\tau, \sigma)$ which we discussed at the beginning of previous section. The simplest example of compactification is one in which only one dimension is curled up forming a microscopic circle of radius R . One can see that the mass² of the string modes has now the form [1] :

$$m^2 = \left(\frac{p^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} \right) + \frac{1}{\alpha'} (N + \tilde{N} - 2) \quad (3)$$

where $p, w = 0, \pm 1, \pm 2, \dots$. In this formula the first term in the rhs. is not special of string theories, it just corresponds to the fact that in quantum mechanics with one dimension of finite size, there are quantized momentum (p) states, the Kaluza-Klein modes in our case. The second term in the rhs. (proportional to R^2) is purely stringy, as is obvious from the fact that α' appears. It represents

the possibility that a closed string winds w times around the circle of radius R . The above formula has an interesting property observed [8] by K. Kikkawa and M. Yamasaki in 1984. It remains invariant under the replacement:

$$p \leftrightarrow w \quad ; \quad R \leftrightarrow \frac{\alpha'}{R}, \quad (4)$$

i.e., we exchange quantized momenta p by winding number w and at the same time R by α'/R . Physically this is quite a surprising symmetry since it indicates that the energies of string states are identical in a theory with compact radius R and in a theory with radius α'/R . It turns out that not only the spectrum but all physical properties of those two string configurations are identical. This is remarkable because this tells us that for a compactified string large R and small R are equivalent! This is the simplest example of what now is called T-duality³. If instead of one dimension of radius R one compactifies two dimensions, the $R \leftrightarrow \alpha'/R$ duality generalizes in an interesting way. The appropriate duality transformation is now [9] :

$$T \rightarrow \frac{1}{T} \quad ; \quad T \rightarrow T + 1 \quad (5)$$

where now T is a complex ("modulus") field $T = b + iR^2$. Here R^2 is the overall compact volume and b is an axion-like field. The above two transformations generate the discrete infinite group $SL(2, Z)$. Again, this transformation has to be accompanied by the appropriate transformation of quantized momenta and winding numbers.

When applied to the list of ten-dimensional closed supersymmetric strings a number of equivalences were found. It turns out that Type IIA string compactified on a circle of radius R is equivalent (T-dual) to Type IIB compactified on a circle of radius α'/R (and viceversa) [10]. Furthermore the heterotic $E_8 \times E_8$ compactified on a circle of radius R is T-dual to the $SO(32)$ heterotic compactified on a circle of radius α'/R (and viceversa) [11]. This is remarkable since it shows us that there are indeed only three types of supersymmetric strings which are disconnected: Type II, Type I and heterotic. Thus the number of candidates for a fundamental theory was substantially reduced. This nevertheless did not impress very much the string practitioners. Tacitly most stringers had in the back of their minds the idea that only the heterotic string had any chance of constituting the long sought for unified theory of all known interactions. Type II and Type I strings were bothersome theories that sooner or later would be proven to be inconsistent. In this way we would be left with a unique unified theory, the heterotic. This turned out not to be the case, as has been shown in the last four years.

³This symmetry was termed just duality. It was first called T-duality in ref.[12] to distinguish it from the newly proposed S-duality.

4 S-duality

In string theory the strength of interactions is not given by coupling constants but by the vacuum expectation value of a massless real scalar ϕ , the dilaton. It turns out that, although apparently totally different in origin, the dilaton behaves to some extent in a way quite similar to the compact radius R which appears in the field T described in the previous section. Let us consider the simplest compactification of the heterotic string down to four dimensions obtained by compactifying on a six torus. The low energy Lagrangian has $N = 4$ supersymmetry in $D = 4$. It turns out that the dilaton appears naturally as the imaginary part of a complex scalar field $S = \eta + i\phi$. Here η is an axion-like field. + Furthermore, at the classical level the field S appears in the Lagrangian in a way quite analogous to the fields T . Both have associated symmetries of the type $SL(2, R)$. We already mentioned above that in the case of the T field a discrete subgroup of that symmetry, $SL(2, Z)$, was in fact exact, corresponding to T -duality. A natural conjecture would then be to assume that, also for the S field an $SL(2, Z)$ symmetry generated by the transformations:

$$S \rightarrow \frac{1}{S} \quad ; \quad S \rightarrow S + 1 \quad (6)$$

could be a symmetry of this class of string vacua and this was our proposal [12] in 1990. But now this symmetry is quite bizarre: notice that eq.(6) includes the transformation $\phi \leftrightarrow 1/\phi$. Since the dilaton ϕ gives a measure of the strength of the interactions, this is a symmetry which relates strongly coupled to weakly coupled string theory! Unlike the case of T -duality this symmetry is clearly non-perturbative in nature and hence difficult to check with purely perturbative methods.

The origin of the $SL(2, Z)$ symmetry in the case of T -duality had to do with the generalization of the simple symmetry $R \leftrightarrow \alpha'/R$ which appears when a closed string has one compact dimension of size R . Is there anything like this in S -duality? In fact it turns out that if one derives the Lagrangian of $D = 10$ supergravity starting from $D = 11$ supergravity and compactifies on a circle of radius R_{11} , the ten-dimensional dilaton $\phi = R_{11}$, and hence one would have hoped that perhaps a duality in R_{11} would have then implied a $\phi \leftrightarrow 1/\phi$ symmetry [12]. But in 1990 $D = 11$ supergravity had no room in the realm of string theory and we had to wait till 1995 to have a geometrical understanding of S -duality.

In the case of T -duality, the transformations of eq.(5) come along with the interchange of quantized momenta p and winding numbers w . What is the equivalent statement in S -duality? We proposed [12] that in the case of S -duality one has to exchange standard charged particles with solitonic monopole states which generically are present in this class of theories. In fact already in 1975 Claus Montonen and David Olive [13] had considered the possibility of an exchange symmetry between charged particles and monopoles in certain

non-supersymmetric gauged field theories with scalars. They conjectured the possibility that the physics of charged particles at strong coupling g^2 could be equivalent to that of magnetic monopoles at small coupling. In the $N = 4$ supersymmetric version of these theories [14] one finds certain states, called BPS states whose masses are given by an expression of the form:

$$M^2 \propto q_e^2 g^2 + \frac{q_m^2}{g^2} \quad (7)$$

where $q_e(q_m)$ are the electric(magnetic) charges of the particles and g^2 is the gauge coupling constant². Notice the explicit invariance under the exchanges $q_e \leftrightarrow q_m$, $g^2 \leftrightarrow 1/g^2$. It is obvious the analogy of this expression to that of the first two terms in the rhs. of eq.(3). In the string case one would have the dilaton ϕ instead of g^2 . Since in the heterotic string compactification that we are discussing there is also an effective $N = 4$ supersymmetric Lagrangian, we considered this as additional circumstantial evidence in favour of the S-duality symmetry [12].

Being a non-perturbative symmetry, it was not obvious how to find evidence in favour of its reality. However A. Sen [15] studied in more detail the heterotic string compactified on the six torus and found that the spectrum of BPS particles was indeed invariant under S-duality. More importantly, starting with a purely electric state, S-duality predicts the existence of certain monopole configurations obtained by $SL(2, Z)$ transformations. Ashoke Sen found those solutions with the correct multiplicities [15]. His arguments applied even non-perturbatively because the mass-formulae for BPS states is necessarily exact due to $N = 4$ supersymmetry. These developments changed the initially skeptical attitude of the community concerning strong-weak coupling dualities and gave rise to the beginning of the so called second revolution of string theory [16]. It also inspired the seminal work of Seiberg and Witten [17] in which they constructed the exact effective Lagrangian of certain gauge field theories with $N = 2$ supersymmetry in four dimensions by using duality arguments.

5 Dualities and unification of all string theories

The S-duality symmetry of the heterotic string compactified on a six-torus which we discussed in the previous section was the first example of a general class of symmetries present in many string configurations. Starting in 1994 many such strong/weak coupling dualities were conjectured to hold. These conjectures have not been formally proven but a number of consistency checks have been made in a good number of cases making difficult to believe that they are purely accidental. It is difficult to briefly describe all the concepts about string theory that have changed in the last four years due to these dualities. We will limit ourselves here to review some of the results which are more directly related to the unification of all string theories into a single structure. In this context

the pioneering work of M. Duff [18] , A. Strominger [19] , P. Townsend and C. Hull [20] , completed and systematized by E. Witten in 1995 [21] were very important.

We already discussed how, due to T-duality, there are only three disconnected ten-dimensional supersymmetric string theories: Type I, Type II and heterotic. The new S-dualities further reduce the number of independent theories. It has been found that [16] :

1. Type I string theory and the heterotic $SO(32)$ are S-dual to each other [22] . This means that ten-dimensional Type I theory at small coupling ϕ is equivalent to heterotic $SO(32)$ at (large) $1/\phi$ coupling. This is in some way the least surprising of all dualities since both theories possess $SO(32)$ gauge bosons.
2. Type IIB theory is S-dual to itself [20] .
3. Type IIA theory gives rise to a surprise [21] . At weak coupling ϕ it is a ten-dimensional theory but increasing the coupling a new eleventh dimension reveals itself. Type IIA theory corresponds to a new theory in eleven dimensions in which one of the dimensions is compactified on a circle with radius $R_{11} = \phi$. The Kaluza-Klein states in this 11-th dimension have masses $\propto 1/\phi$ and that is why at weak coupling (small ϕ) this extra dimension is not seen. This is a new surprising phenomenon: as the strength of interactions increases new dimensions may appear. This 11-dimensional theory, whose low-energy Lagrangian turns out to coincide with that of 11-dimensional supergravity, is termed M-theory and a proper formulation for it is still lacking.
4. Something analogous to the Type IIA case happens for the $E_8 \times E_8$ heterotic [23] . As one increases the coupling ϕ a new eleventh dimension reveals itself with size $R_{11} = \phi$. However in this case the $E_8 \times E_8$ theory is obtained by compactifying the 11-dimensional M-theory on a finite segment of length R_{11} . At each of the two boundary points of the segment one gets a gauge group E_8 and this explains the presence of the two factors.

With all the above connections and equivalences it is clear that all known string theories are connected to each other and to the mysterious 11-th dimensional M-theory: Type IIA and $E_8 \times E_8$ theories are both connected to M-theory at strong coupling. On the other hand T-dualities connect those two theories with Type IIB and $SO(32)$ heterotic respectively. Finally, this last theory is connected by S-duality to Type I $SO(32)$ theory. Thus all five supersymmetric string theories are in one way or another connected to M-theory. All string theories are unified.

6 Dualities and p-branes

An important role in the duality developments is played by p-branes [16, 24]. They are extended objects which are generalizations of the notion of string. We mentioned how the string coordinate $X^\mu(\tau, \sigma)$ depends on one time-like variable τ and a space-like variable σ which parametrizes the position along the string. As the string moves it sweeps a world-sheet parametrized by (τ, σ) . One can as well consider other extended objects (p-branes) whose coordinates $X^\mu(\tau, \sigma_a)$, $a = 1, \dots, p$ depend on more than one space-like variable σ_a . As they move they sweep a $p + 1$ -dimensional "world-volume". Point-like particles are 0-branes, strings are 1-branes, membranes are 2-branes etc. In fact these p-branes appear as soliton-like solutions in the effective low-energy field theories from the different supersymmetric strings. This is very much analogous to monopoles, which also appear as soliton-like solutions of certain gauge theories with scalar fields. In particular, the massless sector of supersymmetric strings contain antisymmetric tensor fields $A_{\mu_1 \dots \mu_n}$. It turns out that one finds p-brane solutions for $p = n - 1$ ("electric") and for $p = D - n - 3$ ("magnetic"), where D is the number of space-time dimensions. Thus, for example, in all $D = 10$ closed strings there is a massless antisymmetric tensor field $B_{\mu\nu}$. Thus $n = 2$ and we have a 1-brane (the fundamental string) and a 5-brane.

In the case of Type II strings in addition one has some extra massless antisymmetric fields with odd(even) number of indices for Type IIA(B). Thus there are p-branes with even(odd) p for Type IIA(B) strings, with $p < 10$. These are called Dirichlet branes [24], D-branes for short, and turn out to be particularly important because they allow for some specific perturbative computations which lead to checks of certain duality conjectures. One important property of these D-branes is that open Type I strings are forced to have their boundary ends lying necessarily on the portion of space occupied by D-branes. In the case of 9-branes their worldvolume occupies all ten dimensions and hence the end-points of an open string may be anywhere. But for $p < 9$ open strings are forced to start and end on submanifolds of the full 10-dimensional space, the submanifold occupied for the relevant D-brane worldvolume. Thus, for example, in the case of 3-branes whose world-volume can be made to coincide with the physical Minkowski space, open strings are forced to start and end on Minkowski space, they cannot start or end on the remaining $10 - 4 = 6$ dimensions (the famous "bulk").

7 Embedding the observed world into D-branes

D-branes have associated gauge fields living in their worldvolume [24]. Due to this fact, open strings starting and ending on the same D-brane give rise to a massless $U(1)$ field. If we have N D-branes with overlapping worldvolume the gauge symmetry is enhanced to $U(N)$. The non-Abelian generators correspond

to open strings going from one D-brane to another. A variety of Type II or Type I string vacua can be constructed containing different p-brane configurations considered as static classical objects. This new class of string vacua may have in general quite large gauge groups, groups with rank much higher than 28 which is the maximum for a $D = 4$ heterotic compactification. This already shows us that the possibilities for embedding the physical $SU(3) \times SU(2) \times U(1)$ group into string theory are much wider than in the perturbative heterotic vacua in which the SM group was always a subgroup either of $E_8 \times E_8$ or $SO(32)$. Thus in principle one can now consider the SM group as coming from a configuration of e.g., four coinciding 3-branes. Although this is true in principle, getting chirality and 3 generations in this manner is not so obvious at the moment. New Type I $D = 4$, $N = 1$ string models have recently been built [25] with the new techniques which incorporate these new features although it is too soon to expect fully realistic models.

If indeed the SM gauge group corresponds to gauge fields living on the world-volume of 3-branes ⁴ some surprises concerning the structure of fundamental mass scales appear [26, 27, 28, 29, 30, 31, 32]. Let me first recall the relationship between Planck mass M_{Planck} and string scale $M_s = 1/\sqrt{\alpha'}$. In the perturbative heterotic string those two scales are unavoidably tied up by the equation $M_s^2 = \alpha_{GUT} M_{Planck}^2 / 8$, where α_{GUT} is the gauge coupling. This is in some way related to the fact that both gravitational and gauge interactions come from closed strings, there is a real unification between both interactions. Thus the string scale is necessarily closed to the Planck mass, $M_s \simeq 10^{17}$ GeV. An important point is that this relationship between M_s and M_{Planck} is (in first approximation) independent of the compactification scale M_c which gives as the overall inverse size of the six extra compact dimensions.

This is not in general the case for Type I string vacua. In this case, if the gauge group comes from open strings starting and ending on a set of p-branes, one has [26, 29, 30, 32] :

$$\frac{M_c^{(p-6)}}{M_s^{(p-7)}} = \frac{\alpha_p M_{Planck}}{\sqrt{2}} \quad (8)$$

where α_p is the gauge coupling of the corresponding gauge group. Consider for example the case in which we assume that our gauge group comes from a set of 3-branes ($p = 3$). Then one has

$$M_s^4 = \frac{\alpha_{GUT}}{\sqrt{2}} M_c^3 M_{Planck} \quad . \quad (9)$$

It is clear from this expression that one can lower the value of the string scale M_s as much as we wish by lowering accordingly the compactification scale M_c and still maintaining M_{Planck} fixed at its experimental value. Thus the standard

⁴Or (q+3)-branes with q dimensions wrapping on the compact dimensions.

statement that string physics happens at the Planck scale is not justified in the present situation. The value of M_s depends on what the size of the extra dimensions are.

One could argue that the compactification scale M_c cannot be below a few hundred GeV, since accelerators have not seen any Kaluza-Klein excitations at those energies. However this conclusion would have been incorrect, as recently emphasized in ref.[28] . Indeed, I already mentioned above that the gauge group coming from 3-branes lives only on Minkowski space since open strings can only start or end on the worldvolume of the 3-branes, which we have identified with Minkowski space. Open strings cannot propagate in the bulk. Other way of saying the same thing is that the gauge group has no Kaluza-Klein excitations. On the other hand the gravitational sector, which comes from closed strings, can propagate in the bulk and has both Kaluza-Klein and winding modes. However this Kaluza-Klein gravitational excitations are very weakly coupled and would not have been observed at accelerators even if M_c was as small as the weak scale.

What is then the scale of string theory M_s ? It cannot be lower than 1 TeV or so since otherwise string excitations would have been observed at accelerators. Notice that although the gauge group has no KK excitations it has string excitations. Thus in principle any value of M_s above 1 TeV is possible. There are however three natural options for the value of M_s which look particularly appealing:

i) $M_s \simeq M_{GUT}$ [26] . Here M_{GUT} is the scale at which the extrapolated gauge couplings of the minimal supersymmetric standard model (MSSM) join. Numerically this is of order 10^{16} GeV. This is compatible with eq.(9) for values of M_c slightly below M_s . This possibility has the advantage that it incorporates the succesfull joining of coupling constants of the MSSM in a natural way. On the other hand the gauge hierarchy between M_W and M_{Planck} has to be blamed to some mechanism (like gaugino condensation) able to generate such large hierarchies.

ii) $M_s \simeq \sqrt{M_W M_{Planck}}$. This is the geometrical intermediate scale $\simeq 10^{11}$ GeV which coincides with the SUSY-breaking scale in models with a hidden sector and gravity mediated SUSY breaking. The interest of this choice has been recently emphasized in ref.[31] . In this case one has $M_c/M_s \simeq \alpha \simeq 0.01$ and the hierarchy between M_W and M_{Planck} may be understood without the necessity of any hierarchy-generating mechanism like gaugino condensation. In addition in this scheme it is easy to accomodate the axion solution to the strong CP problem. Indeed, cosmological and astrophysical constraints imply that the axion scale should be of order of the intermediate scale. On the other hand axion fields with this scale appear naturally in Type I string models with M_s of this order.

iii) $M_s \simeq 1$ TeV [27, 28, 29] . In this case the hierarchy problem is solved in an obvious way by lowering the fundamental scale of gravitation close to the weak scale. In this case eq.(9) gives (for an isotropical compactification)

$M_c/M_s \simeq 10^{-5}$ and hence $M_c \simeq 10$ MeV. As we remarked above, such low values of M_c are perfectly compatible with accelerator data since gauge fields have no KK excitations. In fact one can even consider a non-isotropic case with four compact dimensions with $M_c \simeq 1$ TeV and the other two compact dimensions with $M_c \simeq 10^{-3}$ eV [28, 29]. In this case string physics could be tested at future accelerators such as LHC. On the other hand the large hierarchy between M_c and M_s has now to be explained.

Each of these three possibilities has its own advantages and shortcomings, the first being the most conservative one. The possibility of finding traces of string theory at accelerator energies as may happen in the 1 TeV scenario is quite exciting [33]. On the other hand a number of theoretical issues like proton stability, gauge coupling unification, cosmology and generation of the M_c/M_s hierarchy may be difficult to deal with in that scheme. All those questions seem more easy to deal with in the scheme with M_s equal to the intermediate scale. A lot of effort is at present being dedicated to the study of the different alternatives.

8 Epilogue

The duality revolution has changed not only our ideas about string theory but also about field theory. Many perturbative and non-perturbative properties of supersymmetric gauge theories have been reformulated in the last three years in the language of D-branes giving a geometrical interpretation to facts like the Higgs effect or Seiberg's duality. There are reasons to believe that the D-brane language may lead eventually to a more fundamental formulation of gauge theories themselves. A new duality has also been proposed by Maldacena [34] which relates certain gauge field theories to Type II string theories on certain backgrounds. That has lead even to attempts [35] to compute the glueball spectrum of QCD in terms of the dual Type IIB compactified theory. Although it is not clear whether the approximations made in such computations are in a consistent regime, the mere fact of thinking that one can perhaps compute physical things like glueball masses show how string theory and duality are giving us unexpected tools for the study of field theories.

An important application of D-branes has been to the problem of information loss in blackholes. In particular, for certain classes of blackholes, it has been possible [36] to obtain a microscopic explanation of the Bekenstein-Hawking formula in terms of D-branes (see e.g. refs.[37] for an intuitive explanation of this and references). On the other hand we are still lacking a fundamental formulation of M-theory, the theory which unifies all supersymmetric string theories and 11-dimensional supergravity. There is a candidate for such a fundamental non-perturbative formulation of M-theory which goes under the name of M(atric)-theory [38] (see refs.[37] for an intuitive idea of this). A lot of work remains to be done in order to understand what is the fundamental theory which

incorporates the notion of dualities in a built-in manner. It is sure that many surprises are still to come both from the more theoretical as well as from the more phenomenological sides.

Acknowledgments

I thank the organizers of this meeting for their kind invitation to this wonderful place. And I apologize to those members of the audience who got tangled up in the network of dimensions, dualities and compactifications they were exposed to during my talk.

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